B K BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS

A CBSE DAY-CUM BOYS' RESIDENTIAL SCHOOL

PERIODIC TEST-1 (2026-27)

MATHEMATICS (041)

MARKING SCHEME SET-02

SECTION A

1.	**The number of critical points of** $f(x) = (x-2)^{\frac{2}{3}}(2x+1)$ **a. 2** **b. 0** **c. 1** **d. 3**	**1**
Answer	a. 2	
2.	In case of increasing function, the derivative is a. Greater than or equal to 0 c. 0 b. Less than 0 d. Greater than but not equal to 0	**1**
Answer	a. Greater than or equal to 0	
3.	Let $A = \{0,1,2,3\}$ and let $R = \{(0,0),(0,1),(0,3),(1,0),(1,1),(2,2),(3,0),(3,3)\}$ be a relation in A. then R is a. Reflexive c. Symmetric b. Transitive d. Equivalence	**1**
Answer	a & b	
4.	The function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing in the interval a. $(-\infty, 2) \cup (3, \infty)$ c. $(-\infty, 2)$ b. $(-\infty, 2] \cup [3, \infty)$ d. $(3, \infty)$	
Answer	$b. (-\infty, 2] \cup [3, \infty)$	
5.	A function $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = x^3 + 1.$ Then the function has a. No minimum value c. both maximum and minimum value b. No maximum value d. neither maximum nor minimum values	**1**
Answer	neither maximum nor minimum values	
6.	This is a Assertion and Reason based question. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below. a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). b. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).	**1**

	c. Assertion (A) is true, but Reason (R) is false. d. Assertion (A) is false, but Reason (R) is true. Let $f(x)$ be a polynomial of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$ **Assertion (A) :** $f(x)$ has a minimum at $x = 1$. **Reason (R):** When $\frac{d}{dx}(f(x)) < 0 \ \forall \ x \in (a-h\,, a)$ and $\frac{d}{dx}(f(x)) > 0 \ \forall \ x \in (a\,, a+h)$; where h is an infinitesimally small positive quantity then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.	
Answer	a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	

SECTION – {B}

(This section comprises of very short answer type questions (VSA) of 2 mark each)

7.	Aditi is making a circular dosa . she is spreading the dosa batter such that its radius is increasing at the rate of 2 cm/sec .  Find the rate of change of the area of the dosa ,in terms of π , when the radius is 9 cm .	**2**
	Answer (2 marks): Area of circle: $A = \pi r^2$ ... (1 mark) Differentiating w.r.t time: $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ Substitute $r = 9$, $\frac{dr}{dt} = 2$: $\frac{dA}{dt} = 2\pi \times 9 \times 2 = 36\pi\ \mathrm{cm}^2/\mathrm{sec}$ ... (1 mark) **Final Answer:** $36\pi\ \mathrm{cm}^2/\mathrm{sec}$	

8.	Let L be the set of lines in a plane and R be the relation in L , defined as $R = \{ (L_1, L_2) : L_1 is\ perpendicular\ to\ L_2\}$. Check whether relation is symmetric.	**2**
	1. **Symmetric:** A relation R is symmetric if for every $(L_1, L_2) \in R$, it implies that $(L_2, L_1) \in R$. In this case, if line L_1 is perpendicular to line L_2, then line L_2 is also perpendicular to line L_1. Therefore, R is symmetric. 2. **Reflexive:** A relation R is reflexive if for every line L, $(L, L) \in R$. However, a line cannot be perpendicular to itself. Thus, R is not reflexive. 3. **Transitive:** A relation R is transitive if whenever $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$, it implies that $(L_1, L_3) \in R$. For example, if L_1 is perpendicular to L_2, and L_2 is perpendicular to L_3, it does not necessarily mean that L_1 is perpendicular to L_3. In fact, L_1 and L_3 can be parallel. Therefore, R is not transitive.	

SECTION – {C}

(This section comprises of short answer type questions (SA) of 3 mark each)

9.	For the function $f(x) = x^3 - 6x^2 + 9x - 8$, find the points of local maxima and local minima .	**3**
	Step 1 Find the first derivative of $f(x)$: $f(x) = x^3 - 6x^2 + 9x - 8$ $f'(x) = \frac{d}{dx}(x^3) - \frac{d}{dx}(6x^2) + \frac{d}{dx}(9x) - \frac{d}{dx}(8)$ $= 3x^2 - 12x + 9$ **Step 2** Set $f'(x) = 0$ to find critical points: $3x^2 - 12x + 9 = 0$ Divide both sides by 3: $x^2 - 4x + 3 = 0$	

	Factorize: $(x-1)(x-3)=0$ So, the critical points are: $x=1,\ 3$ **Step 3** Find the second derivative $f''(x)$: $f''(x)=\frac{d}{dx}(3x^2-12x+9)=6x-12$ **Step 4** Evaluate $f''(x)$ at each critical point:	

- At $x = 1$:

$$f''(1) = 6(1) - 12 = 6 - 12 = -6$$

Since $f''(1) < 0$, $x = 1$ is a point of **local maxima.**

- At $x = 3$:

$$f''(3) = 6(3) - 12 = 18 - 12 = 6$$

Since $f''(3) > 0$, $x = 3$ is a point of **local minima.**

Step 5

Find the corresponding y-values:

- At $x = 1$:

$$f(1) = (1)^3 - 6(1)^2 + 9(1) - 8 = 1 - 6 + 9 - 8 = -4$$

- At $x = 3$:

$$f(3) = (3)^3 - 6(3)^2 + 9(3) - 8 = 27 - 54 + 27 - 8 = -8$$

Final Answer

- **Local maximum at** $x = 1$, $f(1) = -4$
- **Local minimum at** $x = 3$, $f(3) = -8$

So, the points of local maxima and minima are:

- **Local maximum:** $(1, -4)$
- **Local minimum:** $(3, -8)$

<table>
<tr><td>10.</td><td>Let $R = \{(m,n)\ ; m,n \in\ \mathbb{N}\ , n\ is\ the\ facor\ of\ m\ (i.e. n|m)\}$ then Check whether R is reflexive, symmetric and transitive.</td><td>3</td></tr>
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1. Check for Reflexivity
A relation is reflexive if every element is related to itself, i.e., $(m, m) \in R$ for all $m \in \mathbb{N}$.
In this case, the condition is $m|m$ (m is a factor of m). Since any natural number m divides itself ($m = m \times 1$), the condition holds.
$$\forall m \in \mathbb{N}, m|m \implies (m, m) \in R$$
Therefore, R is reflexive.

2. Check for Symmetry
A relation is symmetric if $(m, n) \in R$ implies $(n, m) \in R$.
Let's test this with a counter-example. Let $m = 4$ and $n = 2$.
• $2|4$ is true, so $(4, 2) \in R$.
• However, $4|2$ is false (4 is not a factor of 2).
Since $(4, 2) \in R$ but $(2, 4) \notin R$, the relation is not symmetric.

3. Check for Transitivity
A relation is transitive if $(m, n) \in R$ and $(n, p) \in R$ implies $(m, p) \in R$.
1. Assume $(m, n) \in R$, which means $n|m$. Thus, $m = n \cdot k_1$ for some $k_1 \in \mathbb{N}$.
2. Assume $(n, p) \in R$, which means $p|n$. Thus, $n = p \cdot k_2$ for some $k_2 \in \mathbb{N}$.
3. Substitute the second equation into the first:
$$m = (p \cdot k_2) \cdot k_1 = p \cdot (k_2 \cdot k_1)$$
Since $k_2 \cdot k_1$ is a natural number, $p|m$. This means $(m, p) \in R$.
Therefore, R is transitive.

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SECTION – {D}

(This section comprises of Long answer type questions (LA) of 5 mark)

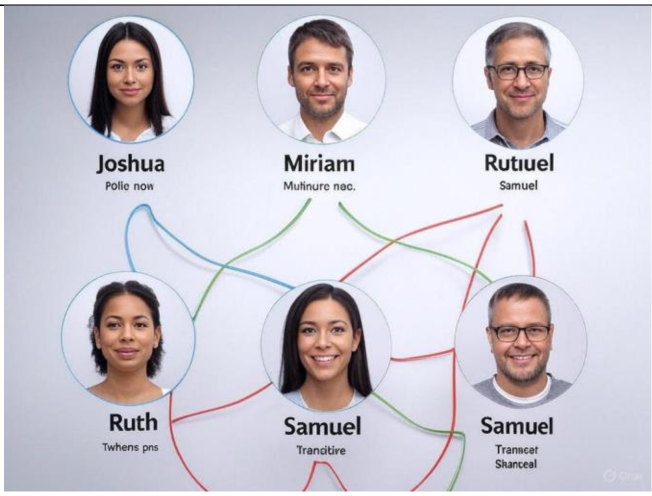
11.	Find the absolute maximum and absolute minimum of function $f(x) = \sin x + \cos x$ on $[0\ ,\pi]$.	**5**

	Given function f(x) = sin x + cos x, on the interval $[0, \pi]$ $f'(x) = \cos x - \sin x$ If $f'(x) = 0$ Then $\cos x - \sin x = 0$ $\therefore \tan x = 1 \Rightarrow x = \frac{\pi}{4}$ At x = 0, $f(0) = \sin 0° + \cos 0° = 1$ At $x = \frac{\pi}{4}, f\left(\frac{\pi}{4}\right) = \sin \ \frac{\pi}{4} + \cos \ \frac{\pi}{4}$ $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ $= \frac{2}{\sqrt{2}}$ $= \sqrt{2}$ At $x = \pi$, $f(\pi) = \sin \pi + \cos \pi$ $= 0 - 1$ $= -1$ $\therefore$ absolute highest value = $\sqrt{2}$ and absolute minimum value = -1	

SECTION – {E}

(This section comprises of CASE BASED questions (CBQ) of 4 mark)

12.	In a social media platform, a group of users P = {Joshua, Miriam, Ruth, Samuel} is analyzed for their connection patterns. A relation R is defined on P where $(x, y) \in R$ means "x follows y." The platform's algorithm records the following connections: • Joshua follows himself (for profile updates). • Miriam follows herself and Joshua. • Ruth follows Ruth and Samuel. • Samuel follows Samuel and Joshua. • Joshua follows Miriam.	**4**



Thus, the relation R is represented as R = {(Joshua, Joshua), (Miriam, Miriam), (Miriam, Joshua), (Ruth, Ruth), (Ruth, Samuel), (Samuel, Samuel), (Samuel, Joshua), (Joshua, Miriam)}.

Based on this case, answer the following question:

1. Check whether the following relation is reflexive .
2. Check whether the following relation is symmetric.
3. Check whether the following relation is transitive .

1. Reflexive

A relation is reflexive if every element is related to itself:

$$(x, x) \in R \text{ for all } x \in P$$

Check:

- (Joshua, Joshua) ✔
- (Miriam, Miriam) ✔
- (Ruth, Ruth) ✔
- (Samuel, Samuel) ✔

✅ **Conclusion:** The relation is **reflexive.**

2. Symmetric

A relation is symmetric if:

$$(x, y) \in R \Rightarrow (y, x) \in R$$

Check key pairs:

- (Miriam, Joshua) and (Joshua, Miriam) ✔
- (Ruth, Samuel) but (Samuel, Ruth) ❌ (not in R)

❌ **Conclusion**: The relation is **not symmetric**.

3. Transitive

A relation is transitive if:

$$(x, y), (y, z) \in R \Rightarrow (x, z) \in R$$

Check:

- (Miriam, Joshua) and (Joshua, Miriam) ⇒ (Miriam, Miriam) ✔
- (Joshua, Miriam) and (Miriam, Joshua) ⇒ (Joshua, Joshua) ✔
- (Ruth, Samuel) and (Samuel, Joshua) ⇒ should have (Ruth, Joshua) ❌ (missing)

❌ **Conclusion:** The relation is **not transitive**.